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Confronting Left-Right Symmetric Models with Electroweak Precision Data at the Z Peak

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ABSTRACT

In view of the recent and future electroweak precision data accumulated at LEP and SLC, we systematically analyze possible new physics effects that may occur in the leptonic sector within the context of $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ theories. It is shown that nonobservation of flavour-violating Z-boson decays, lepton universality in the decays $Z \to l\bar{l}$, and universality of lepton asymmetries at the Z peak form a set of complementary observables, yielding severe constraints on the parameter space of these theories. Contributions of new-physics effects to $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ are found to give interesting mass relations for the flavour-changing Higgs scalars present in these models.

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1 Introduction

The Large Electron Positron collider (LEP) at CERN and the Stanford Linear Collider (SLC) are powerful e^+e^- machines operating at the Z peak, which can confront theoretical predictions of the minimal Standard Model (SM) with experimental results to a high accuracy. A full analysis of all the electroweak precision data including those of the year 1995 will either establish the SM up to one-loop electroweak level or signal the onset of new physics. In this context, analyzing electroweak oblique parameters [1] has become a common strategy to test the viability of models beyond the SM. The electroweak oblique parameters are sensitive physical quantities, when the new-physics interactions couple predominantly to W and Z bosons. However, it is imperative to explore additional observables that could be particularly sensitive to other sectors of the SM.

In this paper, we will study a new set of *complementary* leptonic observables and explicitly demonstrate the severe limitations that can impose on model building of three-generation extensions of the SM. The set of observables comprises flavour-changing leptonic decays of the Z boson [2,3], universality-breaking parameters U_{br} for the diagonal decays $Z \to l\bar{l}$ [4], and universality-violating parameters $\Delta\mathcal{A}_{l_1 l_2}$ based on lepton asymmetries measured at LEP and/or SLC [5]. For the sake of illustration, we will consider a minimal left-right symmetric model (LRSM) [6] described in Section 2. Such a model can naturally generate vector–axial (V–A) as well as V+A flavour-dependent $Zl\bar{l}$ couplings leading to new physics effects that can be probed via the leptonic observables mentioned above. This set of observables will be discussed in some detail in Section 3. In Section 4, we will give numerical estimates of these leptonic observables within the framework of a minimal LRSM and investigate their potential of effectively constraining this model. Furthermore, attention will be paid to possible LRSM contributions to R_b. Section 5 contains our conclusions.

2 The LRSM

Left-right symmetric theories based on the gauge group $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ [7,6] were motivated from the fact that the spontaneous breakdown of gauge and discrete symmetries can be accomplished on the same footing. Such models can naturally

arise from $SO(10)$ grand unified theories via the breaking pattern [7,8]

$$SO(10) \to SU(4)_{PS} \times SU(2)_R \times SU(2)_L \to SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_{B-L} \to \text{SM}.$$

We will, however, focus our analysis on the LRSM described in [6].

In the LRSM, right-handed neutrinos together with the right-handed charged leptons form 3 additional weak isodoublets in a three generation model. The classification of the quark sector proceeds in an analogous way. To be specific, the assignment of quantum numbers to fermions under the gauge group $SU_R(2) \times SU(2)_L \times U(1)_{B-L}$ is arranged as follows:

$$L'_L = \begin{pmatrix} \nu'_l \\ l' \end{pmatrix}_L \ : \ (0, 1/2, -1) \qquad L'_R = \begin{pmatrix} \nu'_l \\ l' \end{pmatrix}_R \ : \ (1/2, 0, -1), \tag{2.1}$$

$$Q'_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \ : \ (0, 1/2, 1/3) \qquad Q'_R = \begin{pmatrix} u' \\ d' \end{pmatrix}_R \ : \ (1/2, 0, 1/3), \tag{2.2}$$

where the prime superscript of the fermionic fields simply denotes weak eigenstates. In order to break the left-right gauge symmetry down to $U(1)_{em}$ [6], we have to introduce one Higgs bidoublet,

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \tag{2.3}$$

that transforms as $(1/2^*, 1/2, 0)$ and two complex Higgs triplets,

$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \quad \text{and} \quad \Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}, \tag{2.4}$$

with quantum numbers $(0, 1, 2)$ and $(1, 0, 2)$, respectively. For simplicity, we will consider that only $\langle \phi_1^0 \rangle = \kappa_1/\sqrt{2}$ and $\langle \delta_R^0 \rangle = v_R/\sqrt{2}$ acquire vacuum expectation values (vev's). In practice, this can be accomplished by imposing invariance of the general Higgs potential under judicious discrete symmetries of the bidoublet Φ and the Higgs triplets $\Delta_{L,R}$ [9]. In fact, we will concentrate on case (d) of Ref. [10], to which the reader is referred for more details. In case (d), it is $\langle \delta_L^0 \rangle = \langle \phi_2^0 \rangle = 0$, implying that the charged gauge bosons W_L and W_R represent also physical states with masses $M_L = M_W$ and M_R, respectively. The massive neutral gauge bosons Z_L and Z_R mix one another with a small mixing angle of order $\kappa_1^2/v_R^2 \sim 10^{-2}$. To a good approximation, we will therefore neglect $Z_L - Z_R$ mixing effects in our calculations.

Such a minimal LRSM allows the presence of baryon–lepton ($B-L$) violating operators in the Yukawa sector. In fact, the $B-L$-violating interactions are introduced by the triplet fields $\Delta_{L,R}$ and give rise to Majorana mass terms $m_{M_{ij}}$ in the following way:

$$\mathcal{L}_{int}^{B-L} = -\frac{\sqrt{2}m_{M_{ij}}}{2v_R}\Big(h_{ij}\bar{L}'^C_{L_i}\,\varepsilon_{ij}\Delta_L L'_{L_j} \;+\; \bar{L}'^C_{R_i}\,\varepsilon_{ij}\Delta_R L'_{R_j}\Big) \quad + \quad \text{H.c.} \tag{2.5}$$

Here, ε_{ij} stands for the usual Levi-Civita tensor and the parameters $h_{ij} = 1$ if left-right symmetry is forced explicitly. However, a phenomenological analysis of muon and τ decays shows that $h_{ij} \ll 1$ [10]. As a result, δ_L^+ and δ_L^{++} loop effects have been found to be negligible [11].

In case (d), the neutrino mass matrix takes the general seesaw-type form

$$M^\nu \;=\; \left(\begin{array}{cc} 0 & m_D \\ m_D^T & m_M \end{array}\right), \tag{2.6}$$

where M^ν is 6×6-dimensional matrix. In Eq. (2.6), m_D is a Dirac mass term connecting the left-handed neutrinos with the right-handed ones. Relevant theoretical and phenomenological aspects related to such neutrino mass models may be found in Ref. [12]. The matrix M^ν can always be diagonalized by a unitary 6×6 matrix U^ν according to the common prescription $U^{\nu T}M^\nu U^\nu = \hat{M}^\nu$. After diagonalization, one gets 6 physical Majorana neutrinos n_i through the transformations

$$\left(\begin{array}{c} \nu'_L \\ \nu'^C_R \end{array}\right)_i \;=\; \sum_{j=1}^{2\mathrm{n}_G} U^{\nu *}_{ij}\; n_{Lj} \quad \text{and} \quad \left(\begin{array}{c} \nu'^C_L \\ \nu'_R \end{array}\right)_i \;=\; \sum_{j=1}^{2\mathrm{n}_G} U^{\nu}_{ij}\; n_{Rj}. \tag{2.7}$$

The first $\mathrm{n}_G = 3$ neutral states, ν_i ($\equiv n_i$ for $i = 1,\ldots,\mathrm{n}_G$), are identified with the known n_G light neutrinos, while the remaining n_G mass eigenstates, N_j ($\equiv n_{j+\mathrm{n}_G}$ for $j = 1,\ldots,\mathrm{n}_G$), are heavy Majorana neutrinos predicted by the model. In addition to the leptonic sector, the quark sector of such an extension contains non-SM couplings of the fermionic fields to the gauge and Higgs bosons. Part of the LRSM couplings has been listed in Ref. [9,10,13]. Relevant Feynman rules and additional discussion is given in Appendix A.

Adopting the conventions of Ref. [12], the interactions of the Majorana neutrinos, n_i, and charged leptons, l_i, with the gauge bosons $W_L^\pm$ ($\equiv W^\pm$) and Z_L are correspondingly mediated by the mixing matrices

$$B^L_{lj} \;=\sum_{k=1}^{\mathrm{n}_G} V^L_{lk}U^{\nu *}_{kj} \quad \text{and} \quad C^L_{ij} \;=\; \sum_{k=1}^{\mathrm{n}_G} U^\nu_{ki}U^{\nu *}_{kj}, \tag{2.8}$$

with $l = 1, \ldots, n_G$ and $i, j = 1, \ldots, 2n_G$. By analogy, there exist mixing matrices B^R_{li} and C^R_{ij} given by

$$B^R_{lj} = \sum_{k=n_G+1}^{2n_G} V^R_{lk} U^{\nu *}_{kj} \quad \text{and} \quad C^R_{ij} = \sum_{k=n_G+1}^{2n_G} U^{\nu}_{ki} U^{\nu *}_{kj}, \tag{2.9}$$

which are responsible for the couplings of $W^\pm_R$ and Z_R to charged leptons and Majorana neutrinos. In Eqs. (2.8) and (2.9), the unitary $n_G \times n_G$-matrices V^L and V^R are responsible for the diagonalization of the charged lepton mass matrix via biunitary transformations.

Due to the specific structure of M^ν in Eq. (2.6), the flavour-mixing matrices B^L and C^L satisfy a number of identities that may be found in [14] These identities, which result from the requirement of unitarity and renormalizability of the theory, turn out to be very useful in deriving model-independent relations between the mixings B^L_{li}, C^L_{ij} and heavy neutrino masses. In a two generation mixing model, we have [15,16]

$$B^L_{lN_1} = \frac{\rho^{1/4} s^{\nu_l}_L}{\sqrt{1+\rho^{1/2}}}\,, \qquad B^L_{lN_2} = \frac{i s^{\nu_l}_L}{\sqrt{1+\rho^{1/2}}}\,, \tag{2.10}$$

where $\rho = m^2_{N_2}/m^2_{N_1}$ (≥ 1) is a mass ratio of the two heavy Majorana neutrinos N_1 and N_2 present in such a model, and $s^{\nu_l}_L$ is L-violating mixings defined as [17]

$$(s^{\nu_l}_L)^2 \equiv \sum_{i=n_G+1}^{2n_G} |B^L_{li}|^2 \simeq \left(m^\dagger_D \frac{1}{m^2_M} m_D \right)_{ll}. \tag{2.11}$$

Furthermore, the mixings $C^L_{N_iN_j}$ are determined by

$$\begin{aligned} C^L_{N_1N_1} &= \frac{\rho^{1/2}}{1+\rho^{1/2}} \sum_{i=1}^{n_G} (s^{\nu_i}_L)^2, \qquad C^L_{N_2N_2} = \frac{1}{1+\rho^{1/2}} \sum_{i=1}^{n_G} (s^{\nu_i}_L)^2, \\ C^L_{N_1N_2} &= -C^L_{N_2N_1} = \frac{i\rho^{1/4}}{1+\rho^{1/2}} \sum_{i=1}^{n_G} (s^{\nu_i}_L)^2. \end{aligned} \tag{2.12}$$

At this stage, it should be noted that M^ν of Eq. (2.6) takes the known seesaw form [18] in case $m_M \gg m_D$. Nevertheless, this mass-scale hierarchy can dramatically be relaxed in a two-family seesaw-type model, which can radiatively induce light-neutrino masses in agreement with experimental upper bounds [12]. The light-heavy neutrino mixings $s^{\nu_l}_L$ of such scenarios can, in principle, scale as $s^{\nu_l}_L \sim m_D/m_M$ rather than the known seesaw relation $s^{\nu_l}_L \sim \sqrt{m_{\nu_l}/m_N}$. This implies that high Dirac components are allowed to be present in M^ν and only the ratio m_D/m_M ($\sim s^{\nu_l}_L$) gets limited by a global analysis of low-energy data. Recently, such an analysis has been performed in Ref. [19], in which the combined

effect of all possible effective operators of the charged- and neutral-current interactions is considered. Although a careful analysis can provide some model-dependent caveats, we will, however, consider the following conservative upper bounds for the L-violating mixings:

$$(s_L^{\nu_e})^2,\ (s_L^{\nu_\mu})^2\ <\ 0.010, \qquad (s_L^{\nu_\tau})^2\ <\ 0.040, \qquad \text{and} \qquad (s_L^{\nu_e})^2(s_L^{\nu_\mu})^2\ <\ 1.\ 10^{-8}. \quad (2.13)$$

For example, the last constraint in Eq. (2.13) comes from the nonobservation of decays of the type $\mu \to e\gamma,\ eee$, or the absence of $\mu - e$ conversion events in nuclei.

In LRSMs, the mixing matrices B^L, C^L, B^R and C^R obey the useful relations

$$\sum_{i=1}^{2\mathrm{n}_G} B^L_{l_1 i} B^R_{l_2 i} = 0, \qquad \sum_{l=1}^{\mathrm{n}_G} B^{R*}_{li} B^R_{lj} = C^R_{ij}, \qquad C^{L*}_{ij}\ +\ C^R_{ij}\ =\ \delta_{ij}. \quad (2.14)$$

In a two-generation mixing model, Eq. (2.14) together with Eq. (2.9) can be used to obtain the mixings

$$\begin{aligned} B^R_{l_1 N_1} &= \cos\theta_R \sqrt{1 - C^L_{N_1 N_1}}, \qquad B^R_{l_2 N_1}\ =\ -\sin\theta_R \sqrt{1 - C^L_{N_1 N_1}} \\ B^R_{l_1 N_2} &= -\cos\theta_R \frac{C^L_{N_1 N_2}}{\sqrt{1 - C^L_{N_1 N_1}}} - i\sin\theta_R \left(\frac{(1 - C^L_{N_1 N_1})(1 - C^L_{N_2 N_2}) - |C^L_{N_1 N_2}|^2}{1 - C^L_{N_1 N_1}} \right)^{1/2}, \\ B^R_{l_2 N_2} &= \sin\theta_R \frac{C^L_{N_1 N_2}}{\sqrt{1 - C^L_{N_1 N_1}}} - i\cos\theta_R \left(\frac{(1 - C^L_{N_1 N_1})(1 - C^L_{N_2 N_2}) - |C^L_{N_1 N_2}|^2}{1 - C^L_{N_1 N_1}} \right)^{1/2}. \end{aligned} \quad (2.15)$$

Consequently, the leptonic sector of this two generation scenario depends only on five free parameters; the masses of the two heavy Majorana neutrinos, m_{N_1} and m_{N_2} [or equivalently m_{N_1} and ρ], the mixing angles $(s_L^{\nu_i})^2$, which are, however, constrained by low-energy data, and an unconstrained Cabbibo-type angle θ_R.

3 SLC and LEP observables

In this section, we will define more precisely the framework of our calculations. In the limit of vanishing charged lepton masses, the amplitude responsible for the decay $Z \to l_1 \bar{l}_2$ can generally be parametrized as

$$\mathcal{T}_l\ =\ \frac{ig_w}{2c_w}\, \varepsilon^\mu_Z\, \bar{u}_{l_1} \gamma_\mu [g_L^{l_1 l_2} \mathrm{P}_L\ +\ g_R^{l_1 l_2} \mathrm{P}_R] v_{l_2}, \quad (3.1)$$

where g_w is the usual electroweak coupling constant, ε_Z^μ is the Z-boson polarization vector, u (v) is the Dirac spinor of the charged lepton l_1 (l_2), $\mathrm{P}_L(\mathrm{P}_R) = (1-(+)\gamma_5)/2$, and $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$. In Eq. (3.1), we have defined

$$g_{L,R}^{l_1l_2} = g_{L,R}^l + \delta g_{L,R}^{l_1l_2}, \qquad g_L^l = \sqrt{\rho_l}(1-2\bar{s}_w^2), \qquad g_R^l = -2\sqrt{\rho_l}\bar{s}_w^2, \tag{3.2}$$

where ρ_l, $\bar{s}_w$, $\delta g_{L,R}^l$ $(\equiv \delta g_{L,R}^{ll})$ are obtained beyond the Born approximation and are renormalization scheme dependent. It should be noted that ρ_l, $\bar{s}_w$ introduce universal oblique corrections [1], whereas $\delta g_{L,R}^{l_1l_2}$ are flavour dependent. Obviously, an analogous expression will be valid for the decay $Z \to b\bar{b}$, as soon as b-quark mass effects can be neglected.

It is convenient to reexpress the flavour-dependent electroweak corrections in terms of the loop functions $\Gamma_{l_1l_2}^L$ and $\Gamma_{l_1l_2}^R$ as follows:

$$\delta g_L^{l_1l_2} \;=\; \frac{\alpha_w}{2\pi}\,\Gamma_{l_1l_2}^L, \qquad \delta g_R^{l_1l_2} \;=\; \frac{\alpha_w}{2\pi}\,\Gamma_{l_1l_2}^R.$$

The nonoblique loop functions $\Gamma_{l_1l_2}^L$ and $\Gamma_{l_1l_2}^R$ depend on whether the underlying theory is of V–A or V+A nature. In Appendix B, we have analytically derived the loop functions $\Gamma_{l_1l_2}^L$ and $\Gamma_{l_1l_2}^R$ in the context of LRSMs. It is then straightforward to obtain the branching ratio for the possible decay of the Z boson into two different charged leptons

$$B(Z \to \bar{l}_1 l_2 + l_1 \bar{l}_2) \;=\; \frac{\alpha_w^3}{48\pi c_w^2}\,\frac{M_Z}{\Gamma_Z}\Big[|\Gamma_{l_1l_2}^L|^2 + |\Gamma_{l_1l_2}^R|^2\Big], \tag{3.3}$$

with $\alpha_w = g_w^2/4\pi$. Such an observable is constrained by LEP results to be, *e.g.*, $B(Z \to e\tau) \lesssim 10^{-5}$ [20].

Another observable that has been introduced in [4] is the universality-breaking parameter $U_{br}^{(l_1l_2)}$. To leading order of perturbation theory, $U_{br}^{(l_1l_2)}$ is given by

$$\begin{aligned} U_{br}^{(l_1l_2)} &= \frac{\Gamma(Z\to l_1\bar{l}_1) \;-\; \Gamma(Z\to l_2\bar{l}_2)}{\Gamma(Z\to l_1\bar{l}_1) \;+\; \Gamma(Z\to l_2\bar{l}_2)} \;-\; U_{br}^{(l_1l_2)}(\mathrm{PS}) \\ &= \frac{g_L^l(\delta g_L^{l_1} - \delta g_L^{l_2}) \;+\; g_R^l(\delta g_R^{l_1} - \delta g_R^{l_2})}{g_L^{l2} \;+\; g_R^{l2}} \\ &= U_{br}^{(l_1l_2)}(\mathrm{LH}) \;+\; U_{br}^{(l_1l_2)}(\mathrm{RH}), \end{aligned} \tag{3.4}$$

where $U_{br}^{(l_1l_2)}(\mathrm{PS})$ characterizes known phase-space corrections coming from the nonzero masses of the charged leptons l_1 and l_2 that can always be subtracted, and

$$U_{br}^{(l_1l_2)}(\mathrm{LH}) \;=\; \frac{g_L^l(\delta g_L^{l_1} - \delta g_L^{l_2})}{(g_L^{l2} + g_R^{l2})} \;=\; \frac{\alpha_w}{2\pi}\frac{g_L^l}{g_L^{l2} + g_R^{l2}}\,\Re e(\Gamma_{l_1l_1}^L - \Gamma_{l_2l_2}^L) \tag{3.5}$$

$$U_{br}^{(l_1l_2)}(\mathrm{RH}) \;=\; \frac{g_R^l(\delta g_R^{l_1} - \delta g_R^{l_2})}{g_L^{l2} + g_R^{l2}} \;=\; \frac{\alpha_w}{2\pi}\frac{g_R^l}{g_L^{l2} + g_R^{l2}}\,\Re e(\Gamma_{l_1l_1}^R - \Gamma_{l_2l_2}^R) \tag{3.6}$$

To make contact with the corresponding observable given in [20], one can easily derive the relation

$$\frac{\Gamma(Z \to l\bar{l})}{\Gamma(Z \to l'\bar{l}')} = 2U_{br}^{(ll')} + 1.$$

The results of a combined analysis at LEP/SLC regarding lepton universality at the Z resonance can be summarized [21,22] as follows:

$$\begin{aligned}
|U_{br}^{(ll')}| &< 5.\ 10^{-3} \qquad (\mathrm{SM}:0), \\
\mathcal{A}_\tau(\mathcal{P}_\tau) &= 0.143 \pm 0.010 \qquad (\mathrm{SM}:0.143), \\
\mathcal{A}_e(\mathcal{P}_\tau) &= 0.135 \pm 0.011, \\
\mathcal{A}_{FB}^{(0,l)} &= 0.0170 \pm 0.0016 \qquad (\mathrm{SM}:0.0153), \\
\mathcal{A}_{LR}(SLC) &= 0.1637 \pm 0.0075.
\end{aligned} \tag{3.7}$$

In parentheses, we quote the theoretical predictions obtained in the SM. From (3.7), we find that the experimental sensitivity to $\Delta\mathcal{A}_{\tau e}$ is about 7%, (4%) for LEP (SLC). Note that $\mathcal{A}_e$ should equal the left-right asymmetry, $\mathcal{A}_{LR}$, measured at SLC. Furthermore, it is worth mentioning that ongoing SLC experiments are measuring the observable

$$A_{LR}^{FB}(f) \;=\; \frac{\Delta\sigma(e_L^- e^+ \to f\bar{f})_{FB} - \Delta\sigma(e_R^- e^+ \to f\bar{f})_{FB}}{\Delta\sigma(e_L^- e^+ \to f\bar{f})_{FB} + \Delta\sigma(e_R^- e^+ \to f\bar{f})_{FB}} = \frac{3}{4}\mathcal{P}_e\mathcal{A}_f, \tag{3.8}$$

The forward-backward left-right asymmetry for individual flavours will be an interesting alternative of establishing possible deviations from SM universality in lepton asymmetries.

Lepton asymmetries [or equivalently forward-backward asymmetries] can also play a crucial rôle to constrain new physics. Here, we will be interested in experiments at LEP/SLC that measure the observable

$$\begin{aligned}
\mathcal{A}_l \;&=\; \frac{\Gamma(Z \to l_L\bar{l}) \;-\; \Gamma(Z \to l_R\bar{l})}{\Gamma(Z \to l\bar{l})} \;=\; \frac{g_L^{ll2} \;-\; g_R^{ll2}}{g_L^{ll2} \;+\; g_R^{ll2}} \\
&=\; \frac{g_L^{l2} - g_R^{l2} + 2(g_L^l\delta g_L^l - g_R\delta g_R^l)}{g_L^{l2} + g_R^{l2} + 2(g_L^l\delta g_L^l + g_R^l\delta g_R^l)}.
\end{aligned} \tag{3.9}$$

In view of the recent discrepancy of more than 2σ between the experimental results of $\mathcal{A}_{LR}$ at SLC and $\mathcal{A}_e$ at LEP, we are motivated to use the nonuniversality parameter of lepton asymmetries [5]

$$\Delta\mathcal{A}_{l_1 l_2} \;=\; \frac{\mathcal{A}_{l_1} \;-\; \mathcal{A}_{l_2}}{\mathcal{A}_{l_1} \;+\; \mathcal{A}_{l_2}} \;=\; \frac{1}{\bar{\mathcal{A}}_l}\Big(U_{br}^{(l_1 l_2)}(\mathrm{LH}) \;-\; U_{br}^{(l_1 l_2)}(\mathrm{RH})\Big) \;-\; U_{br}^{(l_1 l_2)}, \tag{3.10}$$

where $\bar{\mathcal{A}}_l$ may be given by the mean value of the two lepton asymmetries $\mathcal{A}_{l_1}$ and $\mathcal{A}_{l_2}$. At this point, it should be stressed that requiring $U_{br}^{(l_1 l_2)} = 0$ does *not necessarily* imply $\Delta\mathcal{A}_{l_1 l_2} = 0$. As we will later see, in LRSMs one can naturally encounter the possibility, in which $U_{br}(\mathrm{LH}) \simeq -U_{br}(\mathrm{RH})$ while $\Delta\mathcal{A}_{l_1 l_2}$ becomes sizeable. Moreover, the physical quantities $U_{br}^{(l_1 l_2)}$ and $\Delta\mathcal{A}_{l_1 l_2}$ do not depend explicitly on universal electroweak oblique parameters, especially when the latter ones may poorly constrain such three-generation scenarios [4].

Another observable which will still be of interest is

$$R_b \ = \ 0.2202 \pm 0.0020 \qquad (\mathrm{SM} : 0.2158). \tag{3.11}$$

If the measurement at LEP is correct, R_b turns out to be about 2σ off from the theoretical prediction of the minimal SM. New physics contributions to R_b can be conveniently calculated through [23]

$$R_b \ = \ 0.22\Big[1 + 0.78\nabla_b^{(SM)}(m_t) - 0.06\Delta\rho^{(SM)}(m_t)\Big], \tag{3.12}$$

where $\nabla_b^{(SM)}(m_t)$ and $\Delta\rho^{(SM)}(m_t)$ contains the m_t-dependent parts of the vertex and oblique corrections, respectively. Practically, only $\nabla_b^{(SM)}(m_t)$ gives significant negative contributions to R_b, which behave, in the large top-mass limit, as [24]

$$\nabla_b^{(SM)}(m_t) \ \simeq \ -\frac{20\alpha_w s_w^2}{13\pi}\,\frac{m_t^2}{M_Z^2}. \tag{3.13}$$

If there are new physics effects contributing to $\nabla_b^{(SM)}(m_t)$, these can be estimated by

$$\nabla_b^{(new)}(m_t) \ = \ \frac{\alpha_w}{2\pi}\,\frac{g_L^b \Re e(\Gamma_{bb}^L(m_t) - \Gamma_{bb}^L(0)) + g_R^b \Re e(\Gamma_{bb}^R(m_t) - \Gamma_{bb}^R(0))}{g_L^{b2} \ + \ g_R^{b2}}\,, \tag{3.14}$$

where $g_L^b = 1 - 2s_w^2/3$ and $g_R^b = -2s_w^2/3$. In the next section, we will analyze numerically the size of new physics effects expected in LRSM.

4 Numerical results and discussion

Since there is a large number of free parameters that could vary independently in the LRSM, we have fixed to typical values all of them except of one each time and investigated

the behaviour of our observables as a function of the remaining kinematic variable. More explicitly, we have found that δ_R^{++} quantum corrections to the effective $Zl_R\bar{l}_R$ coupling shown in Fig. 1 are very small, since $M_{\delta_R^{++}} > 5$ TeV for phenomenological reasons [10]. The very same lower mass bound should obey the flavour-changing scalars ϕ_2^{0r} $(\equiv \Re e(\phi_2^0)/\sqrt{2})$ and ϕ_2^{0i} $(\equiv \Im m(\phi_2^0)/\sqrt{2})$ [10]. However, the mass difference between the two flavour-changing scalars should not be too large because the latter would lead to large negative contributions to R_b (we will discuss the consequences from a large mass-difference realization between the flavour-changing scalars at the end of this section). In our estimates, we have assumed that ϕ_2^{0r} and ϕ_2^{0i} are nearly degenerate and heavier than 5 TeV. In such a case, loop effects involving flavour-changing scalars are found to be vanishingly small.

In order to increase the predictability of our LRSM but still keep our analysis on a general basis, we shall consider a two-generation mixing scenario. Then, the free parameters of our minimal model are: the lepton-violating mixings $(s_L^{\nu_l})^2$ [which are, however, constrained to some extend by a global analysis of low-energy data], the two heavy neutrino masses m_{N_1} and m_{N_2} [which have been taken to be at the same mass scale m_N], the masses of the charged gauge boson M_R and its orthogonal associate scalar M_h, and a Cabbibo-type angle θ_R that rotates the right-handed charged leptons to the corresponding mass eigenstates.

In Figs. 2(a)–(d), we present plots of $B(Z \to e^-\tau^+ + e^+\tau^-)$ as a function of m_N, M_R, M_h, and θ_R while keeping fixed the remaining kinematic parameters. In Fig. 2(a), we see the characteristic *quadric*, m_N^4/M_W^4, dependence on the branching ratio [3,14]. The dashed, dotted and dash-dotted lines represent results coming purely from the $SU(2)_R$ sector for $(s_L^{\nu_\tau})^2 = 0.040,\ 0.030$, and 0.020, respectively. The solid lines *i*, *ii*, and *iii* correspond to a complete computation for the three different lepton-violating mixings mentioned above. If we assume some typical values for the rest of the parameters, *i.e.* $M_R = 0.4$ TeV, $M_h = 30$ TeV, and $\theta_R = 0$, we find that $B(Z \to e^-\tau^+ + e^+\tau^-) \lesssim 2.\ 10^{-6}$ for $m_N = 3$ TeV. Although the size of new physics effects may be probed at LEP, the reported value is $B(Z \to e\tau) < 10^{-5}$ and it does not yet impose rather severe constraints on the parameter space of the theory. This conclusion is also supported by Figs. 2(b)–(d). In Fig. 2(c), it is worth observing the logarithmic dependence of the mass ratio M_h/M_R on the branching ratio, which can also render the decay channel $Z \to e\tau$ measurable. In Fig. 2(d), one can further see the strong dependence of θ_R on $B(Z \to e^-\tau^+ + e^+\tau^-)$. However, a similar,

though complementary, behaviour will be found to be present in the observables U_{br} and $\Delta\mathcal{A}$.

We are now proceed by examining numerically the dependence of the universality-breaking parameter U_{br} as a function of various kinematic variables shown in Figs. 3(a)–(d). Again, we observe the nondecoupling behaviour of the heavy neutrino mass in the observable U_{br} [4]. The size of new physics becomes significant for $m_N \gtrsim 3$ TeV, *i.e.* $U_{br} \sim 4. - 5.\ 10^{-3}$. In Fig. 3(b), we see that the value of U_{br} decreases rapidly as M_R increases. In Fig. 3(c), we remark again the logarithmic dependence M_h^2/M_R^2 on U_{br}. In our estimates, we have used a Cabbibo-type angle $\theta_R = 45^0$, which turns out to be a rather moderate value as is displayed in Fig. 3(d). As has been mentioned above, the electroweak corrections originating genuinely from the $SU(2)_R$ sector depend on the angle θ_R. Looking at Fig. 3(d), one can readily see that the choice $\theta_R = 45^0$ gives smaller effects of nonuniversality in the leptonic partial widths of the Z boson. If we had chosen $\theta_R = -45^0$, we would have obtained much stronger combined bounds on the mass parameters and L-violating mixings of the LRSM.

One may get the impression that new-physics effects can be minimized by selecting θ_R to lie in a specific range. This is, however, not true, since the universality-breaking parameter $\Delta\mathcal{A}_{l_1l_2}$ will play a complementary rôle as is shown in Figs. 4(a)–(d). In Fig. 4, we list the results after adding both contributions coming from $SU(2)_L$ and $SU(2)_R$ gauge sectors. Thus, we may be sensitive up to $m_N \lesssim 1.5$ TeV for $(s_L^{\nu_\tau})^2 = 0.04$ and $(s_L^{\nu_e})^2 = 0.01$ (see Fig. 4(a)). In Fig. 4(b), we display the decoupling effect of a very heavy W_R. In Fig. 4(c), we find again the logarithmic enhancement caused by the nondegeneracy between $W_R^\pm$ and $h^\pm$. In addition, it should be noted that interesting phenomenology can only arise for relatively light W_R bosons, *i.e.* $M_R \lesssim 1$ TeV. The latter observation can also be verified from Fig. 4(d), in which $\Delta\mathcal{A}$ is drawn as a function of θ_R for $M_R = 0.4,\ 0.6$, and 0.8 TeV. Furthermore, one can easily recognize the complementary rôle that $B(Z \to l_1l_2)$, U_{br}, and $\Delta\mathcal{A}$ play as far as θ_R is concerned, when comparing Figs. 2(d), 3(d), and 4(d). For example, the choice $\theta_R = -45^0$ would make $\Delta\mathcal{A}$ more difficult to observe, whereas U_{br} becomes larger for this value of θ_R. Of course, scenarios where M_R is at the TeV scale may not be compatible with $K_L - K_S$ phenomenology if we assume an exact left-right symmetry in the Yukawa sector of the model. Nevertheless, in LRSMs that possess nonmanifest or pseudomanifest left-right symmetry, such a constraint is not valid any longer [25].

In the following, we will try to address the question whether there exist possibilities of inducing positive contributions to R_b within our LRSM. As has already been noticed in Section 3, only positive contributions to R_b are of potential interest, which will help to achieve a better agreement between theoretical prediction and the experimental value of R_b. In LRSM, we first consider the Feynman graphs 1(m) and 1(n), where the external leptons are replaced by b-quarks and virtual down-type quarks are running in the place of charged leptons. The interaction Lagrangians of the flavour-changing scalars ϕ_2^{0r} and ϕ_2^{0i} with the d, s, b quarks can be obtained by Eq. (A.17) after making the obvious replacements. These couplings are enhanced, as they are proportional to the top-quark mass. In fact, the flavour-changing scalars generate effective $Zb\bar{b}$ couplings of both V–A and V+A nature. In the limit $M_{\phi_2^{0r}},\ M_{\phi_2^{0i}} \gg M_Z$, the effective $Zb\bar{b}$ couplings take the simple form

$$\Re e(\Gamma^R_{bb}(m_t) - \Gamma^R_{bb}(0)) \;=\; \frac{1}{8}|V^R_{tb}|^2 \frac{m_t^2}{M_W^2} \left(\frac{\lambda_S + \lambda_I}{2(\lambda_S - \lambda_I)} \ln\frac{\lambda_S}{\lambda_I} \;-\; 1 \right), \tag{4.1}$$

$$\Re e(\Gamma^L_{bb}(m_t) - \Gamma^L_{bb}(0)) \;=\; -\frac{1}{8}|V^L_{tb}|^2 \frac{m_t^2}{M_W^2} \left(\frac{\lambda_S + \lambda_I}{2(\lambda_S - \lambda_I)} \ln\frac{\lambda_S}{\lambda_I} \;-\; 1 \right), \tag{4.2}$$

where λ_S and λ_I are defined in Appendix B after Eq. (B.1). The analytic function in the parentheses of the r.h.s. of Eqs. (4.1) and (4.2) is always positive and equals zero when the two scalars ϕ_2^{0r}, ϕ_2^{0i} are degenerate. Substituting Eqs. (4.1) and (4.2) into Eq. (3.14), one easily finds that the SM value of R_b is further decreased. This leads automatically to the restriction

$$M_{\phi_2^{0r}} \;\simeq\; M_{\phi_2^{0i}}. \tag{4.3}$$

The mass relation (4.3) has been used throughout our numerical estimates.

Another place that may lead to positive contributions to R_b are due to diagrams similar to Figs. 1(h) and 1(d). Indeed, an analogous calculation gives

$$\Re e(\Gamma^R_{bb}(m_t) - \Gamma^R_{bb}(0)) \;=\; -\frac{1}{4}|V^R_{tb}|^2 \frac{m_t^2}{M_W^2}\, s_\beta^2 c_\beta^2 \left(\frac{\lambda_h + \lambda_R}{2(\lambda_h - \lambda_R)} \ln\frac{\lambda_h}{\lambda_R} \;-\; 1 \right). \tag{4.4}$$

However, the l.h.s. of Eq. (4.4) is proportional to $s_\beta^2 = M_W^2/M_R^2$ yielding a rather small effect. If we insist in cancelling the negative SM vertex correction $\nabla_b^{SM}(m_t)$ through the contribution (4.4), we find the highly unnatural mass ratio

$$\frac{M_h}{M_R} \;\simeq\; e^{70}.$$

The latter also demonstrates the difficulty of obtaining radiatively positive contributions to R_b within the LRSM. In Ref. [26], it has been shown that $Z_L - Z_R$ mixing effects could help in producing contributions of either sign to R_b. We will not pursue this topic here.

5 Conclusions

We have shown that lepton-flavour violating Z-boson decays, lepton universality in the decays $Z \to l\bar{l}$, and universality of lepton asymmetries at LEP/SLC represent a set of complementary observables and can hence impose severe limitations on model-building in the leptonic sector. For our illustrative purposes, we have considered a LRSM with two-generation mixing. We have found that the observables $B(Z \to l_1 l_2)$, $U_{br}^{l_1 l_2}$, and $\Delta\mathcal{A}_{l_1 l_2}$ are sensitive to different parameter-space regions of this minimal scenario. For instance, if $(s_L^{\nu_\tau})^2 = 0.03$, $(s_L^{\nu_e})^2 = 0.01$, $M_R = 0.4$ TeV, and $M_h = 30$ TeV, then heavy neutrinos are found to have masses that do not exceed 2 TeV for any value of the Cabbibo-type angle θ_R. On the other hand, constraints on new physics from R_b prefer scenarios, in which flavour-changing scalars are degenerate in mass.

It may be worth remarking again the fact that LRSMs can naturally predict $U_{br} \simeq 0$, for some choice of parameters, which could naively be interpreted that lepton universality is preserved in nature. As has been shown in this paper, universality violation can manifest itself in lepton asymmetries $\Delta\mathcal{A}_{l_1 l_2}$ as well. This is, however, not an accidental feature of the LRSM but may have a general applicability to unified models, such as supersymmetric extensions of the SM [5]. In general, such theories can naturally generate both nonuniversal V–A and V+A $Zf\bar{f}$-couplings yielding effects that may be detected by current experiments at LEP and SLC.

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A Feynman rules in the LRSM

Although some of the Feynman rules required in our problem were given in Ref. [9,10], we list all the relevant Feynman rules and Lagrangians governing the interactions of the gauge and Higgs bosons with leptons and neutrinos, as well as the trilinear couplings of the bosons. The covariant derivative acts on the Higgs multiplets as follows:

$$
\begin{aligned}
D_\mu \Phi &= \partial_\mu \Phi + i\frac{g_L}{2}\vec{\sigma}\vec{W}_{L\mu}\Phi - i\frac{g_R}{2}\Phi\vec{\sigma}\vec{W}_{R\mu}, && \text{(A.1)}\\
D_\mu \Delta_{R,L} &= \partial_\mu \Delta_{R,L} + i\frac{g_{R,L}}{2}[\vec{\sigma}\vec{W}_{R,L\mu}, \Delta_{R,L}] + ig' B_\mu \Delta_{R,L}, && \text{(A.2)}
\end{aligned}
$$

where σ_i are the known 2×2 Pauli matrices, and g_L (g_R) are the $SU(2)_L$ ($SU(2)_R$) weak coupling constants which will be set equal to $g_w = g_L = g_R$ (g_w is the usual $SU(2)_L$ weak coupling constant in the SM). To facilitate our computational task, we will further assume that the corresponding neutral gauge boson Z_L is the Z of the SM to a good approximation. Also, we will list the novel LRSM interactions together with the SM couplings in order to avoid possible ambiguities between relative signs.

The trilinear couplings of gauge, Higgs, and would-be Goldstone bosons may therefore be obtained by (all momenta flow into the vertex)

$$
\begin{aligned}
Z_{L\nu}(r)W^+_{L\lambda}(p)W^-_{L\mu}(q) &: -ig_w c_w f_{\mu\nu\lambda}(r,q,p), && \text{(A.3)}\\
Z_{L\nu}(r)W^+_{R\lambda}(p)W^-_{R\mu}(q) &: ig_w\frac{s_w^2}{c_w} f_{\mu\nu\lambda}(r,q,p), && \text{(A.4)}\\
Z_{L\nu}(r)G^+_{L}(p)W^-_{L\mu}(q) &: -ig_w M_W\frac{s_w^2}{c_w}\mathrm{g}^{\mu\nu}, && \text{(A.5)}\\
Z_{L\nu}(r)G^+_{R}(p)W^-_{R\mu}(q) &: ig_w M_W\frac{s_\beta^2 - s_w^2}{s_\beta c_w}\mathrm{g}^{\mu\nu}, && \text{(A.6)}\\
Z_{L\nu}(r)h^+(p)W^-_{R\mu}(q) &: -ig_w M_W\frac{c_\beta}{c_w}\mathrm{g}^{\mu\nu}, && \text{(A.7)}\\
Z_{L\mu}(r)G^+_{L}(p)G^-_{L}(q) &: -i\frac{g_w}{2c_w}(1-2s_w^2)(p-q)_\mu, && \text{(A.8)}\\
Z_{L\mu}(r)G^+_{R}(p)G^-_{R}(q) &: -i\frac{g_w}{2c_w}(s_\beta^2-2s_w^2)(p-q)_\mu, && \text{(A.9)}\\
Z_{L\mu}(r)h^+(p)h^-(q) &: -i\frac{g_w}{2c_w}(c_\beta^2-2s_w^2)(p-q)_\mu, && \text{(A.10)}\\
Z_{L\mu}(r)G^\pm_{R}(p)h^\mp(q) &: \mp i\frac{g_w}{2c_w}s_\beta c_\beta(p-q)_\mu, && \text{(A.11)}\\
Z_{L\mu}(r)\phi_2^{0r}(p)\phi_2^{0i}(q) &: \frac{g_w}{2c_w}(p-q)_\mu, && \text{(A.12)}\\
Z_{L\mu}(r)\delta_R^{++}(p)\delta_R^{--}(q) &: 2i\frac{g_w}{c_w}s_w^2(p-q)_\mu. && \text{(A.13)}
\end{aligned}
$$

Here, we have defined $s_\beta = \sqrt{1-c_\beta^2} = M_W/M_R$ and the Lorentz tensor $f_{\mu\nu\lambda}(r,q,p) = (r-q)_\lambda \mathrm{g}_{\mu\nu} + (q-p)_\nu \mathrm{g}_{\lambda\mu} + (p-r)_\mu \mathrm{g}_{\nu\lambda}$.

The corresponding couplings of the gauge, Higgs, and would-be Goldstone bosons to the charged leptons and neutrinos can be read off from the Lagrangians

$$\mathcal{L}_{int}^{W_R} = -\frac{g_w}{\sqrt{2}} W_R^{-\mu}\, B_{li}^R\, \bar{l}\gamma_\mu \mathrm{P}_R n_i \quad + \quad \mathrm{H.c.}, \tag{A.14}$$

$$\mathcal{L}_{int}^{G_R^-} = -\frac{g_w}{\sqrt{2}M_W}\, s_\beta\, G_R^-\, B_{li}^R\, \bar{l}\Big[m_l \mathrm{P}_R - m_{n_i}\mathrm{P}_L\Big] n_i \quad + \quad \mathrm{H.c.}, \tag{A.15}$$

$$\mathcal{L}_{int}^{h^-} = \frac{g_w}{\sqrt{2}M_W}\, c_\beta\, h^-\, \bar{l}\left[B_{li}^R m_l \mathrm{P}_R - B_{lj}^R\left(\delta_{ji} - \frac{C_{ji}^{R*}}{c_\beta^2}\right) m_{n_j}\mathrm{P}_L\right] n_i \quad + \quad \mathrm{H.c.}, \tag{A.16}$$

$$\begin{aligned}\mathcal{L}_{int}^{\phi_2^0} = &-\frac{g_w}{2M_W}\phi_2^{0r}\,\bar{l}_1\Big[B_{l_1j}^L m_{n_j} B_{l_2j}^{R*}\mathrm{P}_R + B_{l_1j}^R m_{n_j} B_{l_2j}^{L*}\mathrm{P}_L\Big] l_2 \\ &-\frac{ig_w}{2M_W}\phi_2^{0i}\,\bar{l}_1\Big[B_{l_1j}^L m_{n_j} B_{l_2j}^{R*}\mathrm{P}_R - B_{l_1j}^R m_{n_j} B_{l_2j}^{L*}\mathrm{P}_L\Big] l_2,\end{aligned} \tag{A.17}$$

$$\mathcal{L}_{int}^{\delta_R^{++}} = \frac{g_w}{2\sqrt{2}M_W}\frac{s_\beta}{c_\beta}\delta_R^{++}\, \bar{l}_1^C\, B_{l_1j}^{R*} m_{n_j} B_{l_2j}^{R*}\mathrm{P}_R\, l_2 \quad + \quad \mathrm{H.c.}, \tag{A.18}$$

where the mixing matrices B^R and C^R are defined in Section 2. The couplings of Z_L ($\equiv Z$) to Majorana neutrinos may be found in Ref. [12].

B The nonoblique $Zl\bar{l}$ vertex

We analytically evaluate the loop amplitudes in the limit of vanishing external lepton masses. We adopt dimensional regularization in conjunction with the reduction algorithm of Ref. [27]. Unlike the metric notation of Ref. [27], we use the Minskowskian metric, $\mathrm{g}^{\mu\nu} = \mathrm{diag}(1,1,...,-1)$.

The nonoblique effective Zll' vertex function is similar to the one obtained in [3]. Its analytic form is given by (summation over repeated indices implied)

$$\begin{aligned}\Gamma_{ll'}^L = B_{li}^L B_{l'i}^{L*}\Bigg\{&\delta_{ij}\Bigg[c_w^2\lambda_Z\Big(C_{11}(\lambda_i,1,1) + C_{23}(\lambda_i,1,1) - C_{22}(\lambda_i,1,1)\Big) \\ &+6c_w^2 C_{24}(\lambda_i,1,1) - s_w^2\lambda_i C_0(\lambda_i,1,1) + \frac{1}{2}(1-2s_w^2)\Big(\lambda_i C_{24}(\lambda_i,1,1) \\ &+\frac{1}{2}\lambda_i B_1(0,\lambda_i,1) + B_1(0,\lambda_i,1)\Big)\Bigg] \\ &+C_{ij}^L\Bigg[-C_{24}(1,\lambda_i,\lambda_j) - \frac{1}{2}\lambda_Z\Big(C_0(1,\lambda_i,\lambda_j) + C_{11}(1,\lambda_i,\lambda_j)\Big)\end{aligned}$$

$$+C_{23}(1,\lambda_i,\lambda_j)-C_{22}(1,\lambda_i,\lambda_j)\Big)-\frac{1}{4}\lambda_i\lambda_j C_0(1,\lambda_i,\lambda_j)\Big]$$
$$+\frac{1}{2}C^{L*}_{ij}\sqrt{\lambda_i\lambda_j}\Big[C_0(1,\lambda_i,\lambda_j)+\frac{1}{2}\lambda_Z\Big(C_{23}(1,\lambda_i,\lambda_j)-C_{22}(1,\lambda_i,\lambda_j)\Big)+C_{24}(1,\lambda_i,\lambda_j)\Big]$$
$$+\frac{1}{4}C^{R}_{ij}\sqrt{\lambda_i\lambda_j}\Big[2C_{24}(0,\lambda_S,\lambda_I)-C_{24}(\lambda_S,0,0)-C_{24}(\lambda_I,0,0)+\frac{1}{2}$$
$$+s_w^2\Big(B_1(0,0,\lambda_S)+B_1(0,0,\lambda_I)\Big)-\frac{1}{2}\lambda_Z\Big(C_{23}(\lambda_S,0,0)-C_{22}(\lambda_S,0,0)$$
$$+C_{23}(\lambda_I,0,0)-C_{22}(\lambda_I,0,0)\Big)\Big]\Big\}, \tag{B.1}$$

where $\lambda_i = m_i^2/M_W^2$, $\lambda_Z = M_Z^2/M_W^2$, $\lambda_S = M^2_{\phi_2^{0r}}/M_W^2$, and $\lambda_I = M^2_{\phi_2^{0i}}/M_W^2$. Note that there is a contribution proportional to C^R_{ij} that originates solely from the Higgs sector of the LRSM. In the notation of [28], the first three of the six arguments of the C functions are always evaluated at $(0,\lambda_Z,0)$.

In the LRSM, virtual neutrinos and Higgs scalars induce a nonuniversal Z boson coupling to right-handed charged leptons, Γ^R, as shown in Fig. 1. The contributions of the individual graphs to Γ^R are listed below

$$\begin{aligned}
\Gamma^R_{ll'}(a) &= B^R_{li}B^{R*}_{l'i}s_w^2\Big[\lambda_Z\Big(C_{22}(\lambda_i,\lambda_R,\lambda_R)-C_{23}(\lambda_i,\lambda_R,\lambda_R)-C_{11}(\lambda_i,\lambda_R,\lambda_R)\Big)\\
&\quad -6C_{24}(\lambda_i,\lambda_R,\lambda_R)\Big], && \text{(B.2)}\\
\Gamma^R_{ll'}(b+c) &= B^R_{li}B^{R*}_{l'i}(s_w^2-s_\beta^2)\lambda_i C_0(\lambda_i,\lambda_R,\lambda_R), && \text{(B.3)}\\
\Gamma^R_{ll'}(d) &= \frac{1}{2}B^R_{li}B^{R*}_{l'i}s_\beta^2(s_\beta^2-2s_w^2)\lambda_i C_{24}(\lambda_i,\lambda_R,\lambda_R), && \text{(B.4)}\\
\Gamma^R_{ll'}(e+f) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}(\delta_{ij}s_\beta^2\lambda_i-\sqrt{\lambda_i\lambda_j}C^L_{ij})\Big(C_0(\lambda_i,\lambda_R,\lambda_h)+C_0(\lambda_j,\lambda_R,\lambda_h)\Big), && \text{(B.5)}\\
\Gamma^R_{ll'}(g) &= \frac{1}{2}B^R_{li}B^{R*}_{l'j}\sqrt{\lambda_k\lambda_n}\left(1-\frac{2s_w^2}{c_\beta^2}\right)(s_\beta^2\delta_{ki}-C^L_{ki})(s_\beta^2\delta_{in}-C^L_{in})C_{24}(\lambda_i,\lambda_h,\lambda_h), && \text{(B.6)}\\
\Gamma^R_{ll'}(h+i) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}\sqrt{\lambda_i\lambda_j}s_\beta^2(s_\beta^2\delta_{ij}-C^L_{ij})\Big(C_{24}(\lambda_i,\lambda_R,\lambda_h)+C_{24}(\lambda_j,\lambda_R,\lambda_h)\Big), && \text{(B.7)}\\
\Gamma^R_{ll'}(j) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}\Big\{C^{L*}_{ij}\Big[1-2C_{24}(\lambda_R,\lambda_i,\lambda_j)-\lambda_Z\Big(C_0(\lambda_R,\lambda_i,\lambda_j)+C_{11}(\lambda_R,\lambda_i,\lambda_j)\\
&\quad +C_{23}(\lambda_R,\lambda_i,\lambda_j)-C_{22}(\lambda_R,\lambda_i,\lambda_j)\Big)\Big]+C^L_{ij}\sqrt{\lambda_i\lambda_j}C_0(\lambda_R,\lambda_i,\lambda_j)\Big\}, && \text{(B.8)}\\
\Gamma^R_{ll'}(k) &= -\frac{1}{4}B^R_{li}B^{R*}_{l'j}s_\beta^2\sqrt{\lambda_i\lambda_j}\Big\{C^L_{ij}\Big[2C_{24}(\lambda_R,\lambda_i,\lambda_j)-\frac{1}{2}+\lambda_Z\Big(C_{23}(\lambda_R,\lambda_i,\lambda_j)\\
&\quad -C_{22}(\lambda_R,\lambda_i,\lambda_j)\Big)\Big]-C^{L*}_{ij}\sqrt{\lambda_i\lambda_j}C_0(\lambda_R,\lambda_i,\lambda_j)\Big\}, && \text{(B.9)}
\end{aligned}$$

$$
\begin{aligned}
\Gamma^R_{ll'}(l) &= -\frac{1}{4}B^R_{lk}B^{R*}_{l'n}\sqrt{\lambda_k\lambda_n}\frac{1}{c^2_\beta}(s^2_\beta\delta_{ki}-C^L_{ki})(s^2_\beta\delta_{nj}-C^L_{jn})\Big\{C^L_{ij}\Big[2C_{24}(\lambda_h,\lambda_i,\lambda_j)-\frac{1}{2} \\
&\quad +\lambda_Z\Big(C_{23}(\lambda_h,\lambda_i,\lambda_j)-C_{22}(\lambda_h,\lambda_i,\lambda_j)\Big)\Big]-C^{L*}_{ij}\sqrt{\lambda_i\lambda_j}C_0(\lambda_h,\lambda_i,\lambda_j)\Big\} \qquad \text{(B.10)} \\
\Gamma^R_{ll'}(m+n) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}C^L_{ij}\sqrt{\lambda_i\lambda_j}C_{24}(0,\lambda_S,\lambda_I), \qquad \text{(B.11)} \\
\Gamma^R_{ll'}(o+p) &= \frac{1}{8}B^R_{li}B^{R*}_{l'j}C^L_{ij}(1-2s^2_w)\sqrt{\lambda_i\lambda_j}\Big[2C_{24}(\lambda_S,0,0)+2C_{24}(\lambda_I,0,0)-1 \\
&\quad +\lambda_Z\Big(C_{23}(\lambda_S,0,0)-C_{22}(\lambda_S,0,0)+C_{23}(\lambda_I,0,0)-C_{22}(\lambda_I,0,0)\Big)\Big], \qquad \text{(B.12)} \\
\Gamma^R_{ll'}(q) &= \frac{1}{2}B^R_{li}B^{R*}_{l'j}C^{R*}_{ij}s^2_w\frac{s^2_\beta}{c^2_\beta}\sqrt{\lambda_i\lambda_j}C_{24}(0,\lambda_\delta,\lambda_\delta), \qquad \text{(B.13)} \\
\Gamma^R_{ll'}(r) &= -\frac{1}{8}B^R_{li}B^{R*}_{l'j}C^{R*}_{ij}s^2_w\frac{s^2_\beta}{c^2_\beta}\sqrt{\lambda_i\lambda_j}\Big[2C_{24}(\lambda_\delta,0,0)-\frac{1}{2}+\lambda_Z\Big(C_{23}(\lambda_\delta,0,0) \\
&\quad -C_{22}(\lambda_\delta,0,0)\Big)\Big], \qquad \text{(B.14)}
\end{aligned}
$$

where $\lambda_h = M^2_{h^+}/M^2_W$ and $\lambda_\delta = M^2_{\delta^{++}_R}/M^2_W$. In addition to the irreducible three-point functions, we should take wave-function renormalization constants into account (Figs. 1(A)–(F)). These additional nonuniversal corrections generated by the selfenergies are calculated to give

$$
\begin{aligned}
\Gamma^R_{ll'}(A) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}s^2_w\Big(1+2B_1(0,\lambda_i,\lambda_R)\Big), \qquad \text{(B.15)} \\
\Gamma^R_{ll'}(B) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}s^2_w s^2_\beta\lambda_i B_1(0,\lambda_i,\lambda_R), \qquad \text{(B.16)} \\
\Gamma^R_{ll'}(C) &= -\frac{1}{2}B^R_{li}B^{R*}_{l'j}\frac{s^2_w}{c^2_\beta}\sqrt{\lambda_k\lambda_n}(s^2_\beta\delta_{ki}-C^L_{ki})(s^2_\beta\delta_{in}-C^L_{in})B_1(0,\lambda_i,\lambda_h), \qquad \text{(B.17)} \\
\Gamma^R_{ll'}(D+E) &= -\frac{1}{4}B^R_{li}B^{R*}_{l'j}C^L_{ij}s^2_w\sqrt{\lambda_i\lambda_j}\Big(B_1(0,0,\lambda_S)+B_1(0,0,\lambda_I)\Big), \qquad \text{(B.18)} \\
\Gamma^R_{ll'}(F) &= \frac{1}{8}B^R_{li}B^{R*}_{l'j}C^{R*}_{ij}s^2_w\frac{s^2_\beta}{c^2_\beta}\sqrt{\lambda_i\lambda_j}B_1(0,0,\lambda_\delta). \qquad \text{(B.19)}
\end{aligned}
$$

The sum of Eqs. (B.2)–(B.19) should be free from UV divergences, when $l \neq l'$. This can easily be verified by employing the identities that the mixing matrices $B^{L,R}$ and $C^{L,R}$ obey (see also discussion in Section 2). An ultimate check for the correctness of our analytic results is the vanishing of all terms involving s^2_w in the limit $\lambda_Z \to 0$, due to electromagnetic gauge invariance.

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Figure Captions

Fig. 1: Feynman graphs contributing to the effective nonoblique $Zl_R\bar{l}_R$ coupling in the LRSM.

Fig. 2: $B(Z \to l_1\bar{l}_2 + \bar{l}_1 l_2, l_1 \neq l_2)$ in a two-generation LRSM as a function of **(a)** the heavy neutrino mass $m_N(= m_{N_1} = m_{N_2})$, **(b)** W_R-boson mass M_R, **(c)** charged Higgs boson M_h, [for the L-violating mixings $(s_L^{\nu_\tau})^2 = 0.04$ (curve-i), 0.03 (curve-ii), 0.020 (curve-iii) and setting $(s_L^{\nu_e})^2 = 0.01$, $(s_L^{\nu_\mu})^2 = 0$], and **(d)** a Cabbibo-type angle θ_R [for $M_R = 0.4$ TeV (curve-i), 0.6 TeV (curve-ii), and 0.8 TeV (curve-iii)]. Numerical estimates coming solely from the $SU(2)_R$ sector are also shown. The results analogous to the curves–i, ii, and iii, are correspondingly given by the dashed, dotted, and dash-dotted lines.

Fig. 3: Numerical estimates of $U_{br}^{l_1 l_2}$ for the same set of parameters as in Fig. 2.

Fig. 4: Numerical estimates of $\Delta\mathcal{A}_{l_1 l_2}$ for the same set of parameters as in Fig. 2. Only the total LRSM contribution to $\Delta\mathcal{A}_{l_1 l_2}$ is shown, where the corresponding curves–i, ii, and iii are now given by the solid, dashed, and dotted lines, respectively.